Worksheet 9 Solution

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1. The manager of a gas station has observed that the times required by drivers to fill their car's tank and pay are quite variable. In fact, the times are exponentially distributed with a mean of 7.5 minutes.

    a. What is the probability that a transaction is completed in less than 5 minutes?

    b. What is the probability that a car can complete the transaction between 5 minutes to 10 minutes?

    c. What is the standard deviation of time until a transaction is completed?

    <u>**Answer:**</u>

    $\mu = 7.5$. We know $\frac{1}{\lambda} = \mu$. Hence, $\lambda = \frac{1}{7.5} = 0.13$. Therefore, $T \sim \exp(\lambda = 0.13)$.

    (a) Using formula in the textbook or slides: $P(T < 5) = 1 - e^{-0.13\times 5} = 0.4779542$.

    (b) Using formula in the textbook or slides: $P(5 < T < 10) = e^{-0.13\times 5} - e^{-0.13\times 10} = 0.249514$.

    (c) For Exponential distribution, $\sigma = \mu = 7.5$.

2. Find the mean and standard deviations:

   a. t_5.

   A: $E(t) = 0$ regardless of degrees of freedom. $V(t) = \frac{5}{5-2} = \frac{5}{3}$. $Sd(t) = \sqrt{\frac{5}{3}} = 1.291$.

   b. χ_9^2.

   A: $E\left(\chi_9^2\right) = 9$. $V\left(\chi_9^2\right) = 2 \times 9 = 18$. $Sd\left(\chi_9^2\right) = \sqrt{18} = 4.243$.

   c. $F_{5,15}$.

   A: $E(F_{5,15}) = \frac{15}{15-2} = \frac{15}{13}$. $V(F_{5,15}) = \frac{2 \times 15^2(5+15-2)}{5(15-2)^2(15-4)} \approx 0.8714363$. $Sd(F_{5,15}) = \sqrt{V(F_{5,15})} = 0.935$.

3. A sample of 40 retirees is drawn at random from a population whose mean age is 72 and standard deviation is 9.

   a. What is the distribution of the sample mean?

   Answer:

   $\bar{X} \rightarrow$ mean age of retirees based on a sample of $n = 40 \rightarrow \bar{X} \sim N(72, \frac{9}{\sqrt{40}}) = N(72, 1.423)$

   b. What is the probability that the mean age of the sample exceeds 73 years old?

   Answer:

$$
\begin{aligned}
P(\bar{X} > 73) &= 1 - P(\bar{X} \leq 73) \\
&= 1 - P(\frac{\bar{X} - 72}{1.423} \leq \frac{73 - 72}{1.423}) && \text{(Standardize to use table)} \\
&= 1 - P(Z \leq 0.70) && \text{(From table, } P(Z < 0.70) = 0.758) \\
&= 1 - 0.7580 = 0.242
\end{aligned}
$$

   c. What is the probability that the mean age of the sample is at most 73 years old?

   Answer:

   Using part b:

$$P(\bar{X} \leq 73) = 1 - P(\bar{X} > 73) = 0.7580$$

   d. What is the probability that the mean age of the sample is between 72 and 75 years old?

   Answer:

$$
\begin{aligned}
P(72 < \bar{X} < 75) &= P(\bar{X} < 75) - P(\bar{X} \leq 72) \\
&= P(\frac{\bar{X} - 72}{1.423} < \frac{75 - 72}{1.423}) - P(\frac{\bar{X} - 72}{1.423} \leq \frac{72 - 72}{1.423}) \\
&= P(Z < 2.11) - P(Z < 0) \\
&= 0.9826 - 0.5 = 0.4826
\end{aligned}
$$

   e. What is the probability that a randomly selected retiree is over 73 years old?

   Answer:

   This question is asking for $P(X > 73)$. We can't answer this question since we don't know the distribution of the population. We only know the distribution of $\bar{X}$, not X itself (later, we will learn how we may be able to estimiate this from data).

4. The amount of time spent by American adults playing Poker is Normally distributed with a mean of 4 hours and standard deviation of 1.25 hours. If four American adults are randomly selected, find the probability that:

   a. their average number of hours spent playing Poker is more than 5 hours per week.

   Answer:

   We know that $\bar{X} \sim N\left(4, \frac{1 \cdot 25}{\sqrt{4}}\right)$. Hence:

$$
\begin{aligned}
P(\bar{X} > 5) &= 1 - P[\bar{X} \leq 5] \\
&= 1 - P\left(\frac{\bar{X} - 4}{0.625} \leq \frac{5 - 4}{0.625}\right) \\
&= 1 - P[Z < 1.6] = 1 - 0.9452 = 0.0548
\end{aligned}
$$

   b. their average number of hours spent playing Poker is between 3 and 6 hours per week.

   Answer:

$$
\begin{aligned}
P(3 < \bar{X} < 6) &= P(\bar{X} < 6) - P(\bar{X} \leq 3) \\
&= P(Z < 3.2) - P(Z < -1.6) \\
&= 0.9993 - 0.0548 = 0.9445
\end{aligned}
$$

   c. all four play Poker for more than 5 hours per week.

   Answer:

$$
P(X > 5) = 1 - P(X \leq 5) = 1 - P(Z \leq 0.8) = 1 - 0.7881 = 0.2119
$$

$$
P(x > 5) \cdot P(x > 5) \cdot P(x > 5) \cdot P(x > 5) = \{P(x > 5)\}^4 = (0.2119)^4 = 0.002
$$

5. To estimate the mean salary for a population of 500 employees, the president of a certain company selected at random a sample of 40 employees.

   a. Would you use the finite population correction factor in calculating the standard error of the sample mean in this case? Explain.

      Answer:

      $\frac{n}{N} = \frac{40}{500} = 0.08 > 0.05 \Rightarrow$ FPC is needed.

   b. If the population standard deviation is $800, compute the standard error both with and without using the finite population correction factor.

      Answer:

      With FPC:

$$\sigma_{\bar{X}} = \sqrt{\frac{N-n}{N-1}} \cdot \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{500-40}{500-1}} \cdot \frac{800}{\sqrt{40}} = 121.4475$$

      Without FPC:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{800}{\sqrt{40}} = 126.4911$$