Worksheet 7 Solution

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1. 5 students are giving a make-up quiz. The probability of any of them scoring more than 25 is 0.6. Let X be the number of students who get over 25.

   a. Identify the distribution of X and its parameters.

   b. What is the probability that none of the students score over 25 ?

   c. What is the probability that at least one of them score over 25 ?

   d. What is the probability that all of them score over 25 ?

   Answer:

   (a) $X \sim \text{Bin}(n = 5, p = 0.6)$.

   (b) Using table in the textbook: $P[X = 0] = P[X \leqslant 0] = 0.0102$.

   Or you can calculate directly. $P[X = 0] = 5C0\ p^0(1-p)^{5-0} = 0.1024$.

   (c) $P(X \geqslant 1) = 1 - P(X < 1) = 1 - P(X \leqslant 0) = 1 - 0.0102 = 0.9898$.

   (d) $P(X = 5) = P(X \leqslant 5) - P(X \leqslant 4) = 1 - 0.9222 = 0.0788$.

   Or (again) you can calculte directly: $P[X = 5] = 5C5\ p^5(1-p)^{5-5} = 0.07776$

2. A six-sided die is rolled 6 times. Let X denote the number of times an even number showed up.

   a. What is the probability of the event happening? That is, the probability of getting an even number.

   b. What distribution will X follow? Identify the parameters.

   c. Calculate $P[X = 2]$.

   d. Calculate $P[0 \leq X < 3]$

   Answer:

(a) That is, the probability of getting an even number. $P(x) = \frac{1}{2}$ (for a single roll of the die).

(b) $X \sim \text{Bin}(n = 6, p = 0.5)$

(c) $P[x = 2] = P[x \leqslant 2] - P[x \leqslant 1] = 0.3438 - 0.1094 = 0.2344$

(d)

$$\begin{aligned} P[0 \leqslant x < 3] &= P[x \leqslant 2] - P[x < 0] = P[x \leqslant 2] = P[x < 3] \\ &= 0.3438 \end{aligned}$$

3. Acme Corporation's helpdesk gets 4 calls per day on average. They think the number of calls follows a Poisson distribution.

   a. What is the probability that they get 3 calls or less on a given day?

   b. What is the probability that they get no calls on given day?

   c. What is the probability that they get exactly 3 calls?

   d. What is the expected number of calls in a week?

   e. What is the standard deviation for calls in a day?

Answer:

(a) Using table: $P[x \leqslant 3] = 0.4335$.

(b) Using table:

$$P[x = 0] = P[x \leqslant 0] = 0.0183$$

Direct calculation:

$$P[x = 0] = \frac{e^{-4}4^0}{0!} = 0.01831564$$

(c) Using table: $P[x = 3] = P[x \leqslant 3] - P[x \leqslant 2] = 0.4335 - 0.2381 = 0.1954$.

Direct calculation: $P[x = 3] = \frac{e^{-4}4^3}{3!} = 0.1953668$

(d) Let $Y = \#$ calls per week on average. Then, $Y = 7X$. $E(Y) = E(7X) = 7E(X) = 7 \times 4 = 28$.

(e) $V(X) = \mu = 4 \rightarrow SD(x) = \sqrt{4} = 2$.

4. The number of flaws in an optic fiber cable follows a Poisson Distribution. The average number of flaws in 50 m is 1.5. Let x = number of flaws in 50 m.

   a. What is the probability of exactly 2 flaws in 100 m ?

   b. What is the probability of 3 flaws or less in 150 m ?

   Answer:

   (a) Let Y = number of flaws in 100 m. $Y = 2X$. $E(Y) = 2E(X) = 3$. Using table:

$$P(Y = 2) = P(Y \leqslant 2) - P(Y \leqslant 1) = 0.4232 - 0.1991 = 0.2241$$

   Or, using direct calculation:

$$P(Y = 2) = \frac{e^{-3}3^2}{2!} = 0.2240418$$

   (b) Let Z = number of flaws in 150 m. $Z = 3X$. $E(Z) = 3E(X) = 4.5$. Using table:

$$P(Y \leqslant 3) = 0.3423$$

5. X is a Uniformly distributed random variable that has maximum and minimum values of 10 and 50.

   a. What is the density function and its graph?

   b. Calculate the mean of X.

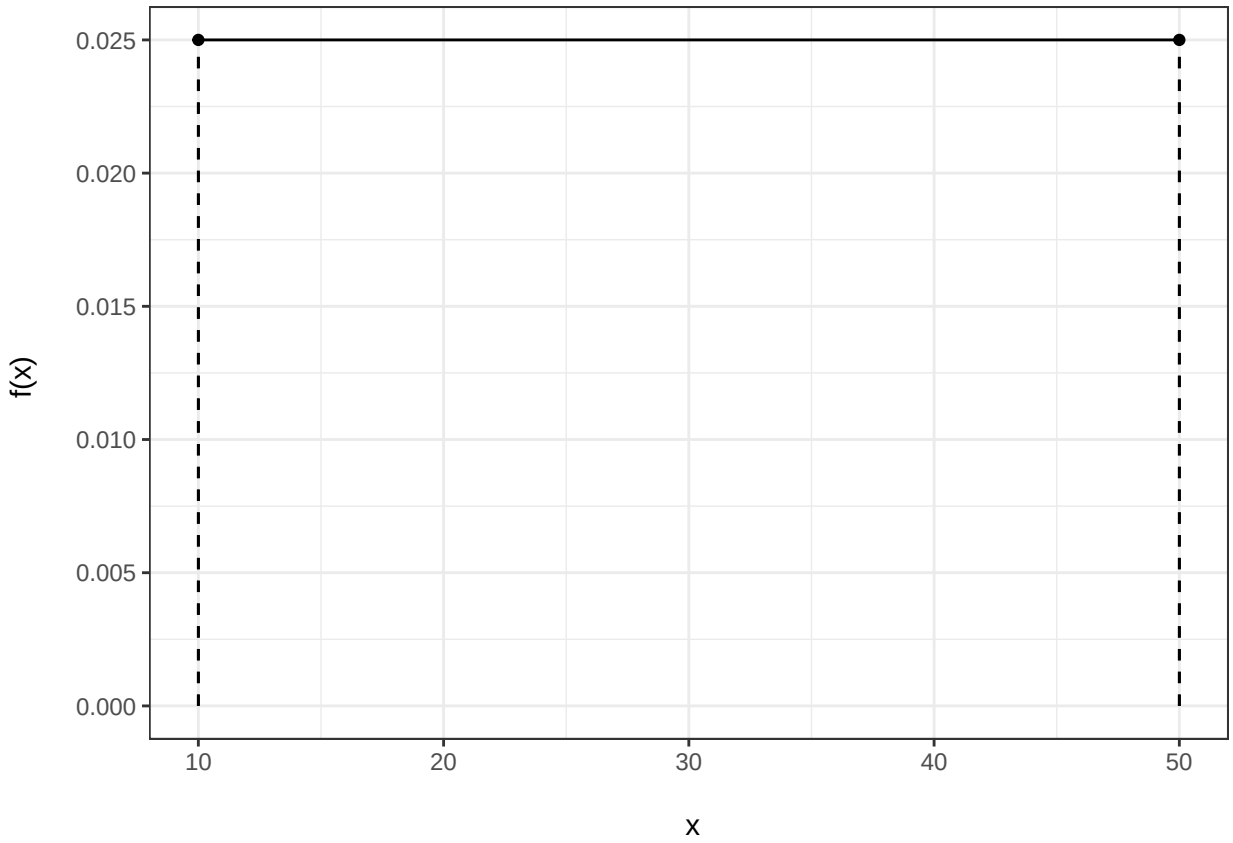
   c. Calculate $P[10 < X < 20]$ and mark the corresponding area on the graph from (a).

   d. Calculate $P[25 < X < 45]$ and mark the corresponding area on the graph from (a).

   e. Find $P[X = 22.5]$.

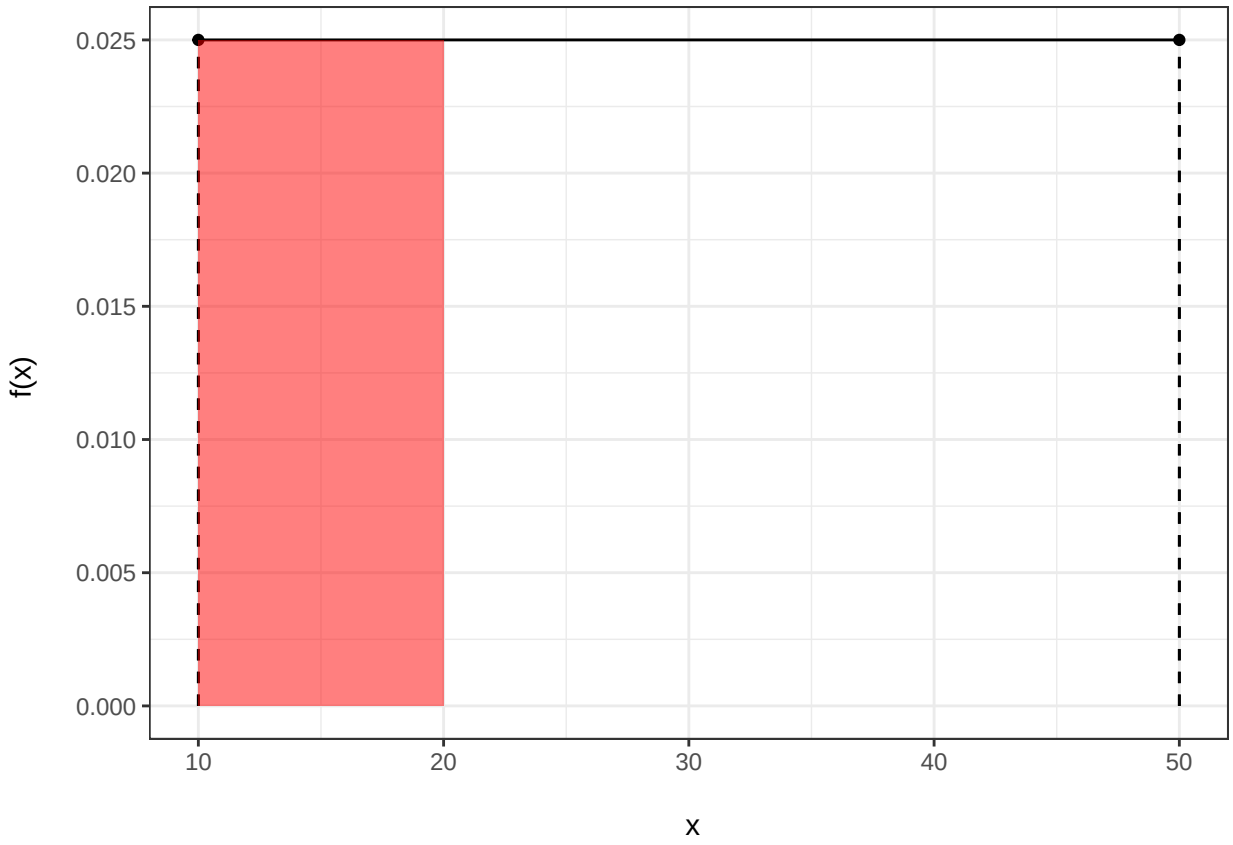
   Answer:

   (a) $f(x) = \frac{1}{b-a} = \frac{1}{50-10} = \frac{1}{40}$.

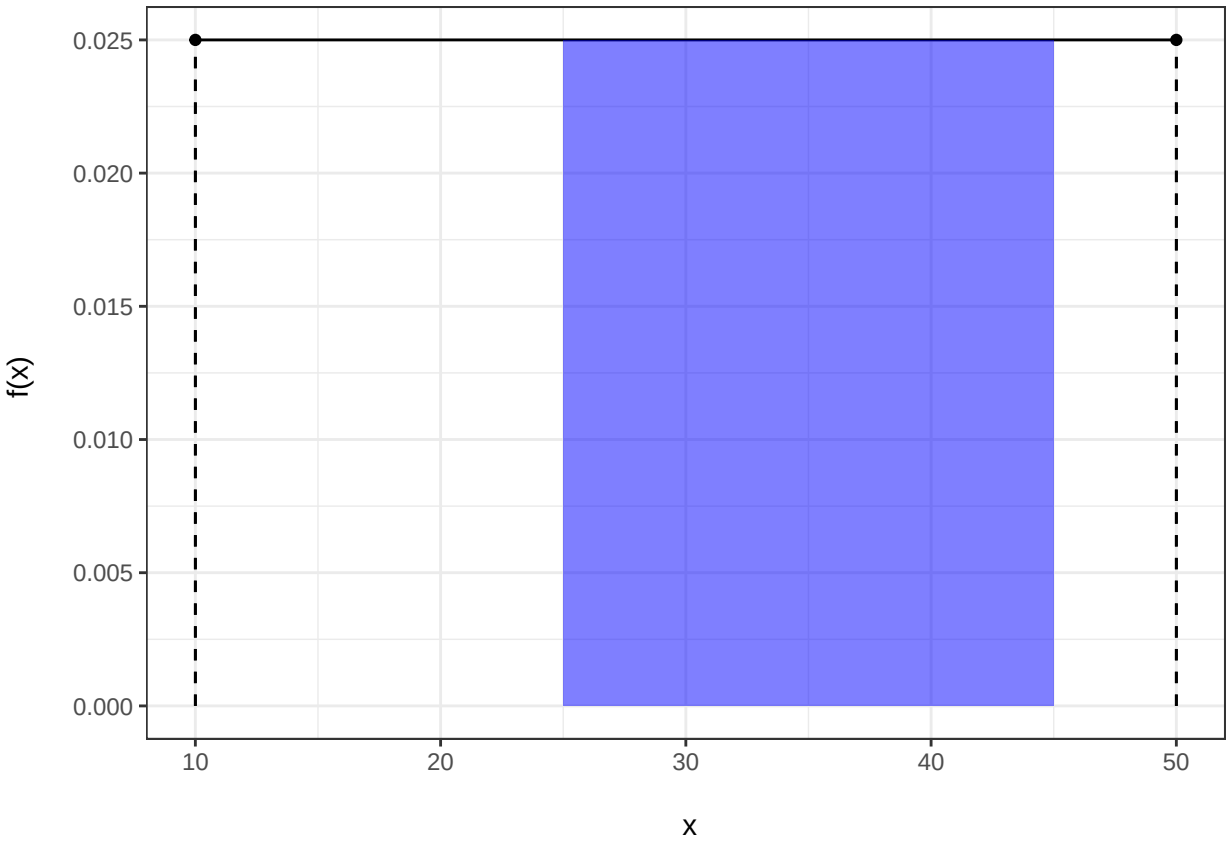


(b) $\mu = \frac{a+b}{2} = \frac{10+50}{2} = 30$.

(c) Red area$= (20-10) \times \frac{1}{40} = \frac{1}{4} = 0.25$.



(d) blue area$= (45-25) \times \frac{1}{40} = \frac{1}{2} = 0.5$.



(e) This is a continuous RV. Hence, $P[X = 22.5] = 0$.