

Worksheet 6 Solution

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1. The number of pizzas delivered to university students each month is a random variable with the following probability distribution.

X	0	1	2	3
P(x)	0.1	0.3	0.4	0.2

- (a) Find $P(X \leq 2)$.
- (b) $P(1 \leq X \leq 2)$.
- (c) Determine the mean and variance of X .
- (d) Suppose $Y = 3X + 4$ for each value of X , Calculate the mean and sd of Y .

Answer:

(a) $P(X \leq 2) = P(x = 0) + P(x = 1) + P(x = 2) = 0.1 + 0.3 + 0.4 = 0.8$

(b) $P(1 \leq x \leq 2) = P(x = 1) + P(x = 2) = 0.7$

(c)

$$\begin{aligned}\mu = E(x) &= \sum x \cdot P(x) = 0 \times 0.1 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.2 = 1.7 \\ V(x) &= \sum (x - \mu)^2 \cdot P(x) = (0 - 1.7)^2 \times 0.1 + (1 - 1.7)^2 \times 0.3 + (2 - 1.7)^2 \times 0.4 \\ &\quad + 3 \times (3 - 1.7)^2 \times 0.2 = 0.81\end{aligned}$$

(d)

$$\begin{aligned}E(y) &= E(3x + 4) = 3E(x) + 4 = 3 \times 1.7 + 4 = 9.1 \\ V(y) &= V(3x + 4) = 9V(x) + 0 = 7.29 \\ sd(y) &= \sqrt{V(y)} = \sqrt{7.29} = 2.7\end{aligned}$$

2. The number of persons living per household in a city was collected and were summarized. The frequency table is given below:

Number of persons	1	2	3	4	5	6	7
Number of households (millions)	3.1	4.5	3.8	2.5	2.4	2.5	1.2

Define X as the number of people per household.

- (a) Write down the probability distribution of X .
- (b) Find $P[X \geq 4]$
- (c) Find $P[2 \leq X < 4]$

Answer:

- (a)

X	1	2	3	4	5	6	7
$P(x)$	0.155	0.225	0.19	0.125	0.12	0.125	0.06

(b) $P[X \geq 4] = P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) = 0.125 + 0.12 + 0.125 + 0.06 = 0.43$

(c) $P[2 \leq X < 4] = P(x = 2) + P(x = 3) = 0.225 + 0.19 = 0.415$

3. Indicate whether each random variable is Binomial, Poisson or neither:
- A random variable X counts the number of visits to a webpage in a one-hour period. **Poisson**
 - A random variable X counts the number of defectives in a sample of 100 stamps. **Binomial**
 - A random variable X counts the number of hearts drawn from a well shuffled deck of 52 playing cards if 10 cards were drawn one at a time without replacement. **Neither**
 - A random variable X counts the number of hearts drawn from a well shuffled deck of 52 playing cards if 10 cards were drawn with replacement. **Binomial**
4. 5 students are giving a make-up quiz. The probability of any of them scoring more than 25 is 0.6. Let X be the number of students who get over 25.
- Identify the distribution of X and its parameters.
 - What is the probability that none of the students score over 25 ?
 - What is the probability that at least one of them score over 25 ?
 - What is the probability that all of them score over 25 ?

Answer:

(a) $X \sim \text{Bin}(n = 5, p = 0.6)$.

(b) Using table in the textbook: $P[X = 0] = P[X \leq 0] = 0.0102$.

Or you can calculate directly. $P[X = 0] = {}^5C_0 p^0(1-p)^{5-0} = 0.1024$.

(c) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X \leq 0) = 1 - 0.0102 = 0.9898$.

(d) $P(X = 5) = P(X \leq 5) - P(X \leq 4) = 1 - 0.9222 = 0.0778$.

Or (again) you can calculate directly: $P[X = 5] = {}^5C_5 p^5(1-p)^{5-5} = 0.07776$