Worksheet 5 Solution

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1. 4 candidates are running for mayor; Adams, Brown, Collins and Dalton (We assume one of the candidates is going to win, there is no run off). The following probabilities are assigned:

$$P[\text{Adams wins}] = 0.42 \quad P[\text{Brown wins}] = 0.09$$

$$P[\text{Collins wins}] = 0.27 \quad P[\text{Dalton wins}] = 0.22$$

Determine the probabilities for the following events (use 2 decimal places):

a. Adams loses.

Answer:

Use the complement Rule.

P[Adams lose] = 1 - P[Adams wins] $= 1 - 0.42 = 0.58$.

b. Either Brown or Dalton wins.

Answer:

Note that only one can win, therefore probability of both of them winning (P(brown win $\cap$ Dalton win)=0). Use the addition rule:

$$P[\text{Brown win}] + P[\text{Dalton win}] = 0.09 + 0.22 = 0.31$$

c. Adams, Brown, or Collins wins.

Answer:

Two options. First one is the use of complement rule:

P[Adams wins or Brown wins or Collins wins] $= 1 - P$ [Dalton win] $= 1 - 0.22 = 0.78$

Or user addition rule considering that probability of all intersections are zero.

P[Adams wins or Brown wins or Collins wins]= P[Adams wins]+P[Brown wins]+P[Collins wins]=$0.42 + 0.09 + 0.27 = 0.78$.

2. $P[A] = 0.30$ and $P[B] = 0.40$. If A and B are mutually exclusive events, what is $P[A \cup B]$?

Answer:

Since A and B are mutually exclusive events, then $P(A \cap B) = 0$. Using the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - 0 = 0.30 + 0.40 = 0.7.$$

3. $P[A] = 0.60$ and $P[B] = 0.70$. If A and B are independent events, what is $P[A \cup B]$?

   Answer:

   Since A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$. Using the addition rule again:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.6 + 0.7 - .42 = 0.88$$

4. Assume you have applied for two scholarships, a Merit scholarship (M) and an Athletic scholarship (A). The probability that you receive an Athletic scholarship is 0.18. The probability of receiving both scholarships is 0.11. The probability of getting at least one of the scholarships is 0.3. (i.e. $P(A) = 0.18$, $P(A \cap M) = 0.11$, and $P(A \cup M) = 0.3$.)

   (a) What is the probability that you will receive a Merit scholarship?

   Answer:

$$P(A \cup M) = P(A) + P(M) - P(A \cap M)$$

$$\Rightarrow 0.3 = 0.18 + P(M) - 0.11 \Rightarrow P(M) = 0.23$$

   (b) Are events A and M mutually exclusive? Why or why not? Explain.

   Answer:

   $P(A \cap M) \neq 0 \Rightarrow A$ and M are not mutually exclusive.

   (c) Are the two events A, and M, independent? Explain, using probabilities.

   Answer:

   $P(A \cap M) = 0.11$. But $P(A) \cdot P(M) = 0.0414$. Therefore, $P(A \cap M) \neq P(A) \cdot P(M)$. Hence A and M are not independent.

   (d) What is the probability of receiving the Athletic scholarship given that you have been awarded the Merit scholarship?

   Answer:

$$P(A \mid M) = \frac{P(A \cap M)}{P(M)} = \frac{0.11}{0.23} = 0.4783$$

5. 60% of the student body at UTC is from the state of Tennessee (T), 30% are from other states (O), and the remainder constitutes international students (I). Twenty percent of students from Tennessee lives in the dormitories, whereas, 50% of students from other states live in the dormitories. Finally, 80% of the international students live in the dormitories.

(a) What percentage of UTC students lives in the dormitories?

(b) Given that a student lives in the dormitory, what is the probability that they are an international student?

(c) Given that a student does not live in the dormitory, what is the probability that they are an international student?

Answer:

(a)

$P(T) = 0.6, P(O) = 0.3, P(I) = 0.1.$

$P(D \mid T) = 0.2 \rightarrow P(D^c \mid T) = 1 - P(D \mid T) = 0.8.$ $P(D \mid O) = 0.5 \rightarrow P(D^C \mid O) = 0.5.$ And $P(D \mid I) = 0.8 \rightarrow P(D^C \mid I) = 0.2.$

$$\begin{aligned} P(D) &= P(D \cap T) + P(D \cap O) + P(D \cap I) \\ &= P(D \mid T) \times P(T) + P(D \mid O) \times P(O) + P(D \mid I) \times P(I) \\ &= 0.2 \times 0.6 + 0.5 \times 0.3 + 0.8 \times 0.1 \\ &= 0.12 + 0.15 + 0.08 = 0.35 \end{aligned}$$

(b) $P(I \mid D) = \frac{P(I \cap D)}{P(D)} = \frac{0.8 \times 0.1}{0.35} = 0.229$. Note that for $P(I \cap D)$, we used what we had in the calculation of part a.

Alternatively, using Bayes rule: $P(I \mid D) = \frac{P(D|I) \cdot P(I)}{P(D)} = \frac{0.8 \times 0.1}{0.35} = 0.229.$

(c)

$$\begin{aligned} P(I \mid D^c) = \frac{P(I \cap D^c)}{P(D^c)} &= \frac{P(D^c \mid I) \cdot P(I)}{P(D^c)} = \frac{P(D^c \mid I) \cdot P(I)}{1 - P(D)} \\ &= \frac{0.2 \times 0.1}{0.65} = 0.0308 \end{aligned}$$

6. A survey asked 100 residents in a town whether they are smokers. Given the following information on the residents' response:

	Daily Workout	No daily workout	Total
Non-smoker	40	30	70
Smoker	20	10	30
Total	60	40	100

(a) Find the joint probability table.

(b) What is the probability that a randomly selected resident worksout daily?

(c) What is the probability that a randomly selected resident doesn't workout daily?

(d) What is the probability that a randomly selected resident worksout daily and is a smoker?

(e) What is the probability that a randomly selected resident worksout daily or is a smoker?

(f) A randomly selected resident doesn't workout daily. What is the probability that the resident is a smoker?

<u>Answer:</u>

(a)

	DW	NDW	Total
NS	0.4	0.3	0.7
S	0.2	0.1	0.3
Total	0.6	0.4	1

(b) $P(DW) = 0.6$.

(c) $P(NDW) = 1 - P(DW) = 0.4$

(d) $P(DW \cap S) = 0.2$

(e)

$$\begin{aligned} P(DW \cup S) = P(DW) + P(S) - P(DW \cap S) &= 0.6 + 0.3 - 0.2 \\ &= 0.7 \end{aligned}$$

(f) $P(S|NDW) = \frac{P(S \cap NDW)}{P(NDW)} = \frac{0.1}{0.4} = 0.25$