Worksheet 11 Solution

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1. A battery manufacturer claims their product has a life expectancy of 90 hours. An improvement production process is believed to make an increase in the life expectancy of batteries. A sample of 36 batteries showed an average life of 93 hours. Assume from past information, the standard deviation of the life expectancy is 9 hours.

   a. Formulate the hypotheses for this problem.

   Answer:

   $$H_0 : \mu \leq 90$$

   $$H_1 : \mu > 90$$

   b. Calculate the test statistic.

   Answer:

   $$\begin{array}{l} \bar{x} = 93 \\ \sigma = 9 \\ n = 36 \end{array} \quad z_{\text{calc}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{93 - 90}{9/\sqrt{36}} = 2$$

   c. Make a decision at 0.1 significance level. State your conclusion in terms of the problem.

   Answer:

   $$c = |z_\alpha| = |z_{0.1}| = 1.28 \quad |z_{calc}| > c \Rightarrow \text{ Reject } H_0.$$

   At 10% level of significance, there is statistical evidence to suggest that the population mean is greater than 90.

   d. Make a decision at 0.01 significance level. What is your conclusion.

   Answer:

$$c = |z_\alpha| = |z_{0.10}| = 2.33 \quad |z_{calc}| < c \Rightarrow \text{Fail to reject } H_0.$$

At 1% level of significance, there isn't enough statistical evidence to suggest that the population mean is greater than 90.

2. The average gasoline price of one of the major oil companies is $1.75 per gallon. Because of cost reduction measures, it is believed that there has been a significant reduction in the average price. In order to test this belief, a sample of 36 of the company's gas stations were randomly selected and yielded an average price $1.65 per gallon. Assume the standard deviation of the population is $0.12.

   a. State the null and the alternative hypothesis for this problem.

   Answer:

$$H_0 : \mu \geqslant 1.75$$

$$H_1 : \mu < 1.75$$

   b. Calculate the test statistic.

   Answer:

   $\bar{x} = 1.65 \quad \sigma = 0.12 \quad n = 36.$

$$z_{\text{calc}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1.65 - 1.75}{0.12/\sqrt{36}} = -5$$

   c. At 5% level of significance, test the company's claim.

   Answer:

$$c = |z_\alpha| = |z_{0.05}| = 1.645.$$

   $\mid z_{calc} \mid > c \Rightarrow$ Reject H_o.

   At 5% level of significant, there is statistical evidence lo suggest that that population mean is less than $1.75.

3. To determine the average price of hotel rooms in Atlanta, a sample of 49 hotels was selected and yielded an average price of hotel rooms being $120. The population standard deviation was found to be $16.

   a. Formulate the hypotheses to determine whether the average price of hotel rooms is significantly different from $124.5.

   Answer:

$$H_0 : \mu = 124.5$$

$$H_a : \mu \neq 124.5$$

b. Calculate the test statistic.

Answer:

$$\bar{x} = 120 \quad n = 49 \quad \sigma = 16$$
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{120 - 124.5}{16/\sqrt{49}} = -1.97$$

c. At 10% level of significance, use critical value approach to test the hypotheses. What is the conclusion?

Answer:

$c = \left|Z_{\alpha/2}\right| = |Z_{0.05}| = 1.645$

$|Z| > C \Rightarrow$ Reject H_o. At 10% level of significant, there is statistical evidence to suggest that the population mean in different from 124.5.

d. At 90% confidence, using the confidence interval approach to test the hypotheses. What is the conclusion?

Answer:

$$1 - \alpha = 0.9 \Rightarrow \alpha/2 = 0.05 \Rightarrow z_{\alpha/2} = 1.645$$
$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 3.76.$$

Hence, 90% CI for population mean $= \bar{x} \pm E$

$$= 120 \pm 3.76 = (116.24, 123.76)$$

since the 90% CI for true population mean does not include the H_o value of 124.5, there is statistical evidence to suggest that mean of the population is different from 124.5.