Worksheet 9

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1. The manager of a gas station has observed that the times required by drivers to fill their car's tank and pay are quite variable. In fact, the times are exponentially distributed with a mean of 7.5 minutes.

   a. What is the probability that a transaction is completed in less than 5 minutes?

   b. What is the probability that a car can complete the transaction between 5 minutes to 10 minutes?

   c. What is the standard deviation of time until a transaction is completed?

$$T \sim Exp\left(\lambda = \frac{1}{\mu}\right) \sim Exp\left(\lambda = \frac{1}{7.5} \approx 0.13\right)$$

(a) $P(T < 5) = 1 - e^{-0.13 \times 5} \cong 0.48$

(b) $P(5 < T < 10) = e^{-0.13 \times 5} - e^{-0.13 \times 10}$

$\cong 0.25$

(c) $\sigma = \mu = 7.5$

2. Find the mean and standard deviations:

a. t_5. $\longrightarrow$ $E(t_5) = 0$ $\quad Var(t_5) = \frac{5}{5-2} = \frac{5}{3}$

$sd = \sqrt{Var} = \sqrt{5/3}$

b. χ^2_9.

c. $F_{5,15}$.

$E(\chi^2_9) = 9$

$Var(\chi^2_\nu) = 2 \cdot 9 = 18$

$sd = \sqrt{Var} = \sqrt{18} \approx 4.23$

$\nu_2 = 15 \longrightarrow E(F_{5,15}) = \frac{15}{15-2} = \frac{15}{13}$

$$Var(F_{5,15}) = \frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)} = \frac{2(15)^2(5+15-2)}{5(15-2)^2(15-4)}$$

≈ 0.87

$sd = \sqrt{Var} = \sqrt{0.87} \approx 0.93$

3. A sample of 40 retirees is drawn at random from a population whose mean age is 72 and standard deviation is 9.

   a. What is the distribution of the sample mean?

   b. What is the probability that the mean age of the sample exceeds 73 years old?

   c. What is the probability that the mean age of the sample is at most 73 years old?

   d. What is the probability that the mean age of the sample is between 72 and 75 years old?

   e. What is the probability that a randomly selected retiree is over 73 years old?

$\underbrace{n=40}$, $\mu = 72$ $\sigma = 9$

$$\text{CLT} \xrightarrow{40>30} \bar{x} \sim N\left(72, \frac{\sigma}{\sqrt{n}} \approx 1.423\right)$$

(σ = 9, n = 40)

(b) $P(\bar{X} > 73) = P\left(\frac{\bar{x}-\mu}{1.423} > \frac{73-72}{1.423}\right)$

$= P(Z > 0.7) = 1 - Pr(Z < 0.7) = 1 - 0.7580 = 0.242$

(c) $P(\bar{X} < 73) = P(Z < 0.7) = 0.7580$

(d) $P(72 < \bar{X} < 75) = \underbrace{P(\bar{X} < 75)} - P(\bar{X} < 72)$

$= P\left(Z < \frac{75-72}{1.423}\right) - Pr\left(Z < \underbrace{\frac{72-72}{1.423}}_{0}\right)$

$= Pr(Z < 2.11) - Pr(Z < 0) = 0.9826 - 0.5 = 0.4826$

(e) by info in the question

→ No Answer

4. The amount of time spent by American adults playing Poker is Normally distributed with a mean of 4 hours and standard deviation of 1.25 hours. If four American adults are randomly selected, find the probability that:

   a. their average number of hours spent playing Poker is more than 5 hours per week.

   b. their average number of hours spent playing Poker is between 3 and 6 hours per week.

   c. all four play Poker for more than 5 hours per week.

$\checkmark\ \boxed{T \sim N(4, 1.25)} \longrightarrow 5 \sim N \longrightarrow \bar{X} \sim N$

$\xrightarrow{5}$ $n = 4$ randomly $\to \bar{X}_4 \sim N\left(4, \frac{1.25}{\sqrt{4}}\right)$ 0.625

(a) $Pr(\bar{X} > 5) = Pr\left(Z > \frac{5-4}{0.625}\right) = Pr(Z > 1.6)$

$= 1 - Pr(Z \leq 1.6) = 1 - 0.9452 = 0.0548$

(b) $Pr(3 < \bar{X} < 6) = Pr(\bar{X} < 6) - Pr(\bar{X} < 3)$

$= Pr\left(Z < \frac{6-4}{0.625}\right) - Pr\left(Z < \frac{3-4}{0.625}\right)$

$= Pr(Z < 3.2) - Pr(Z < -1.6) = 0.9993 - 0.0548$

$= 0.9445$

$Pr(X_1 > 5)\ Pr(X_2 > 5)\ Pr(X_3 > 5)\ Pr(X_4 > 5)$

$Pr(X_1 > 5) = Pr\left(Z > \frac{5-4}{\underbrace{1.25}_{0.8}}\right) = 1 - Pr(Z < 0.8)$

$= 1 - 0.7881$

$= 0.2119$

Pr(All 4 players Play > 5)

$= (0.2119)^4$

5. To estimate the mean salary for a population of 500 employees, the president of a certain company selected at random a sample of 40 employees.

   a. Would you use the finite population correction factor in calculating the standard error of the sample mean in this case? Explain.

   b. If the population standard deviation is \$800, compute the standard error both with and without using the finite population correction factor.