$P(1 < X \le 2) = P(X = 2)$

$P(1 \le X < 2) = P(X = 1)$

Worksheet 6

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2023-03-14

1. The number of pizzas delivered to university students each month is a random variable with the following probability distribution.

$1 - \underbrace{P(X > 2)}_{P(X=3)}$

X	0	1	2	3
P(x)	0.1	0.3	0.4	0.2

   (a) Find P(X ≤ 2). $= P(X=0) + P(X=1) + P(X=2) = 0.1 + 0.3 + 0.4$
   $= 0.8$

   (b) P(1 ≤ X ≤ 2). $= P(X=1) + P(X=2)$
   $= 0.3 + 0.4 = 0.7$

   (c) Determine the mean and variance of X.

   (d) Suppose $Y = 3X + 4$ for each value of X, Calculate the mean and sd of Y.

$E(X) = \sum_{\underbrace{\text{all } x}_{0,1,2,3}} x\, P(X) = 0 * 0.1 + 1 * 0.3 + 2 * 0.4 + 3 * 0.2$

$= 0 + 0.3 + 0.8 + 0.6 = 1.7$

$Var(X) = \sum_{\text{all } x} (x - 1.7)^2 P(X)$

$Var(X) = \underbrace{E(X^2)}{} - [E(X)]^2$

$= 0^2 * 0.1 + 1^2 * 0.3 + 2^2 * 0.4 + 3^2 * 0.2 - [1.7]^2$

$= 0 + 0.3 + 1.6 + 1.8 - [1.7]^2$

$= 3.7 - 2.89 = 0.81$

$sd = \sqrt{729} = 27$

ⓓ $E(3X+4) = 3E(X)+4 = 3 * 1.7 + 4 = 9.1$

$Var(3X+4) = Var(3X) = 9 Var(X) = 9 * 0.81 = 7.29$

2. The number of persons living per household in a city was collected and were summarized. The frequency table is given below:

Number of persons	1	2	3	4	5	6	7
Number of households (millions)	3.1	4.5	3.8	2.5	2.4	2.5	1.2

Define X as the number of people per household.

(a) Write down the probability distribution of X.

(b) Find $P[X \geq 4]$

(c) Find P[2 ≤ X < 4]

Practice

3. Indicate whether each random variable is Binomial, Poisson or neither:

fixed interval time

visit ≡ S

a. A random variable X counts the number of visits to a webpage in a one-hour period. → Poisson

b. A random variable X counts the number of defectives in a sample of 100 stamps. → Binomial

$n = 100$

c. A random variable X counts the number of hearts drawn from a well shuffled deck of 52 playing cards if 10 cards were drawn one at a time without replacement. → neither

H

d. A random variable X counts the number of hearts drawn from a well shuffled deck of 52 playing cards if 10 cards were drawn with replacement.

Heart $\boxed{\frac{13}{52}}$ → $S = \frac{12}{51}$, $F = \frac{13}{51}$ → Not binomial exper

not heart → $1 - \frac{13}{52}$

→ $\boxed{\text{Not Poission experim}}$

$n = 10$ ✓ H / not H ✓

$\frac{13}{52}$ ✓

→ Binomial distribution (assuming very well shuffling.

4. 5 students are giving a make-up quiz. The probability of any of them scoring more than 25 is 0.6. Let X be the number of students who get over 25.

$n=5$ $\begin{cases} \text{Score} \geq 25 \rightarrow 0.6 \\ \text{not} \geq 25 \rightarrow \end{cases}$

   a. Identify the distribution of X and its parameters.

$\sim \text{Bin}(\boxed{5}, 0.6)$

   b. What is the probability that none of the students score over 25 ?

$P(X=x) = nCx\, p^{x} (1-p)^{n-x}$

   c. What is the probability that at least one of them score over 25 ?

   d. What is the probability that all of them score over 25 ?

(b) $P(X=0) = \underbrace{5C0}_{1} \underbrace{(0.6)^{0}}_{1} (1-0.6)^{5-0} = (0.4)^{5} = 0.01024$

0.0102

5. A six-sided die is rolled 6 times. Let X denote the number of times an even number showed up.

   a. What is the probability of the event happening? That is, the probability of getting an even number.

   b. What distribution will X follow? Identify the parameters.

   c. Calculate $P[X = 2]$.

   d. Calculate $P[0 \leq X < 3]$

6. Acme Corporation's helpdesk gets 4 calls per day on average. They think the number of calls follows a Poisson distribution.

   a. What is the probability that they get 3 calls or less on a given day?

   b. What is the probability that they get no calls on given day?

   c. What is the probability that they get exactly 3 calls?

   d. What is the expected number of calls in a week?

   e. What is the standard deviation for calls in a day?

7. The number of flaws in an optic fiber cable follows a Poisson Distribution. The average number of flaws in 50 m is 1.5. Let x = number of flaws in 50 m.

   a. What is the probability of exactly 2 flaws in 100 m ?

   b. What is the probability of 3 flaws or less in 150 m ?