Worksheet 5

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1. 4 candidates are running for mayor; Adams, Brown, Collins and Dalton (We assume one of the candidates is going to win, there is no run off). The following probabilities are assigned:

$$P[\text{Adams wins}] = 0.42 \quad P[\text{Brown wins}] = 0.09$$

$$P[\text{Collins wins}] = 0.27 \quad P[\text{Dalton wins}] = 0.22$$

Determine the probabilities for the following events (use 2 decimal places):

a. Adams loses. → $P(\text{Adam loses}) = 1 - P(\text{adam wins})$
$= 1 - 0.42$
$= 0.58$

b. Either Brown or Dalton wins.

c. Adams, Brown, or Collins wins.

$P(BW \cup DW)$

addition Rule $= P(BW) + P(DW) - P(BW \cap DW)$

$= 0.09 + 0.22 = 0.31$

(c) $P(AW \cup BW \cup CW) = P(H) + P(CW) - P(H \cap CW)$

$= P(AW) + P(BW) + P(CW) = 0.42 + 0.09 + 0.27$

$= 0.78$

2. $P[A] = 0.30$ and $P[B] = 0.40$. If A and B are mutually exclusive events, what is $P[A \cup B]$?

3. $P[A] = 0.60$ and $P[B] = 0.70$. If A and B are independent events, what is $P[A \cup B]$?

addition Rule : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(2)

$= 0.3 + 0.4 - 0$

$= 0.7$

(3)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A)\,P(B)$

$= 0.6 + 0.7 - 0.6 \cdot 0.7$

$= 0.88$

4. Assume you have applied for two scholarships, a Merit scholarship (M) and an Athletic scholarship (A). The probability that you receive an Athletic scholarship is 0.18. The probability of receiving both scholarships is 0.11. The probability of getting at least one of the scholarships is 0.3.

   a. What is the probability that you will receive a Merit scholarship?

   b. Are events A and M mutually exclusive? Why or why not? Explain. $P(A \cap M) \neq 0$ NO

   c. Are the two events A, and M, independent? Explain, using probabilities.

   d. What is the probability of receiving the Athletic scholarship given that you have been awarded the Merit scholarship? $P(A|M) = ? = \frac{P(A \cap M)}{P(M)} = \frac{0.11}{0.23} = 0.47$

   e. What is the probability of receiving the Merit scholarship given that you have been awarded the Athletic scholarship? $P(M|A) = \frac{P(A \cap M)}{P(A)} = \frac{0.11}{0.18} \simeq 0.61$

$P(A) = 0.18$, $P(A \cap M) = 0.11$

$P(A \cup M) = 0.3$

(a) $P(M) = ?$

$P(A \cup M) = P(A) + P(M) - P(A \cap M)$

$0.3 = 0.18 + P(M) - 0.11$

$-0.18 \quad -0.18 \quad +0.11$

$P(M) = 0.23$

(c) $P(A \cap M) \overset{??}{=} P(A)P(M)$

$0.11 \overset{??}{=} 0.18 \times 0.23$

$0.11 \neq 0.04$ → Not independent

5. 60% of the student body at UTC is from the state of Tennessee (T), 30% are from other states (O), and the remainder constitutes international students (I). Twenty percent of students from Tennessee lives in the dormitories, whereas, 50% of students from other states live in the dormitories. Finally, 80% of the international students live in the dormitories.

   (a) What percentage of UTC students lives in the dormitories?

   (b) Given that a student lives in the dormitory, what is the probability that they are an international student?

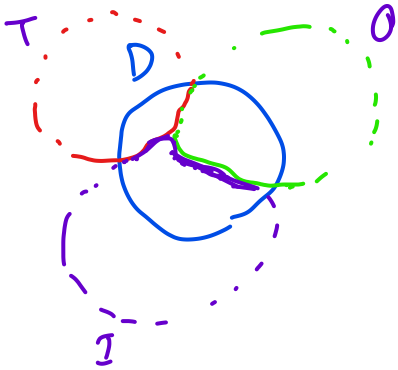
   (c) Given that a student does not live in the dormitory, what is the probability that they are an international student?

$P(T) = 0.6 \quad P(O) = 0.3 \quad P(I) = 0.1$

$P(D|T) = 0.2 \quad P(D|O) = 0.5 \quad P(D|I) = 0.8$

(a) $P(D) = P(D \cap T) + P(D \cap I) + P(D \cap O)$

$= P(D|T)P(T) + P(D|I)P(I) + P(D|O)P(O)$



$= 0.2 * 0.6 + 0.8 * 0.1 + 0.5 * 0.3$

$= 0.12 + 0.08 + 0.15 = 0.35$

(b) $P(I|D) = \frac{P(I \cap D)}{P(D)} = \frac{P(D|I) \cdot P(I)}{P(D)}$

$= \frac{0.8 * 0.1}{0.35} \simeq 0.23$

$(1 - P(D|I))$

(c) $P(I|D^c) = \frac{P(D^c|I)\,P(I)}{P(D^c)}$ $\quad P(D^c) = 1 - P(D)$

$0.8 \qquad 0.1$

$$= \frac{(1 - P(D|I)) \cdot P(I)}{1 - P(D)}$$

$P(D) = 0.35$

6. A survey asked 100 residents in a town whether they are smokers. Given the following information on the residents' response:

	Daily Workout	No daily workout	Total
Non-smoker	40	30	70
Smoker	20	10	30
Total	60	40	100

(a) Find the joint probability table.

(b) What is the probability that a randomly selected resident worksout daily?

(c) What is the probability that a randomly selected resident doesn't workout daily?

(d) What is the probability that a randomly selected resident worksout daily and is a smoker?

(e) What is the probability that a randomly selected resident worksout daily or is a smoker?

(f) A randomly selected resident doesn't workout daily. What is the probability that the resident is a smoker?