Chapter 6 Review

Fred Azizi

2023-10-17

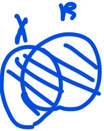
Quick review (1)

Given a sample space $S = \{O_1, O_2, \ldots, O_k\}$

- $0 \leq P(O_i) \leq 1$ for each i
- $\sum_{i=1}^{k} P(O_i) = 1$.

Quick review (2)

- Mutually exclusive: No two outcomes can occur at the same time.
- Exhaustive events: All possible outcomes are included.
- Intersection of Events A and B: the event that occurs when both A and B occur. $P(A \cap B)$



- Union of Events A and B is the event that occurs when either A or B or **both** occur. It is denoted as A or B. $P(A \vee B)$
- Conditional Probability: The probability of event A given event B is

$$P(A \mid B) = \frac{P(A \overset{\cap}{\text{and}} B)}{P(B)}$$

- Independent Events: A and B are said to be independent if $\frac{P(A \cap B)}{P(B)} = P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$.

$\rightarrow \boxed{P(A \cap B) = P(A)\,P(B)}$

Quick review (3)

- Complement Rule: $P\left(A^C\right) = 1 - P(A)$.
- Multiplication Rule: $P(A \text{ and } B) = P(B)P(A \mid B)$.
- Addition Rule: $P(A \underset{\cup}{\text{or}} B) = P(A) + P(B) - P(A \underset{\cap}{\text{and}} B)$

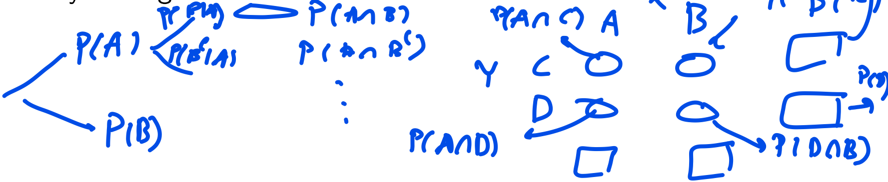
Quick review (4)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

$$P(B \mid A)\,P(A) + P(B \mid A^c)\,P(A^c)$$

- Bayes Rule:
  $P(A_i \mid B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1)+P(A_2)P(B|A_2)+\cdots+P(A_k)P(B|A_k)}$.
- Probability Trees: Helps with showing the events in an experiment by lines using conditionality.
- Joint Probabilities table: Shows the joint probabilities across different levels of categories.
- Marginal probabilities: From Joint distribution table, computed by adding across rows or down columns.

P(C)



Quick review (5)

$P(A)$ $P(B)$

Additional terminologies (for homework only):

For question 14 of homework we define: C = an individual has the genetic condition, $\mathrm{C^c}$ = an individual does not have the genetic condition, PT = a positive blood test, and NT = a negative blood test.

- $P(PT \mid C)$ is called the **sensitivity**.
- $P\left(NT \mid C^C\right)$ is called the **specificity**
- $P(C \mid PT)$ is called the **positive predictive value**.
- $P\left(C^C \mid NT\right)$ is called the **negative predictive value**.

$P(C)$ is given at the beginning of the question. Use **Bayes Rule** to find positive predictive value and negative predictive value

$$P(C \mid PT) = \frac{P(C \cap PT)}{P(PT)} = \frac{P(PT \mid C) \cdot P(C)}{P(PT \mid C)P(C) + P(PT \mid C^c)P(C^c)}$$